Goal Adaptive Discretization of a One-Dimensional Boltzmann Equation

Wijnand Hoitinga

Fluid flow problems in the transitional continuum/molecular regime play an important role in many engineering applications. Moreover, such problems are gaining further prominence with the perpetual trend towards miniaturization in science and engineering. The numerical simulation of flows in the transitional molecular/continuum regime and the determination of *macroscale quantities* from such simulation poses a fundamental challenge, on account of the complexity of the corresponding model equations.

The model equation that describes a flow in the transitional molecular/continuum regime is given by the *Boltzmann equation*. The Boltzmann equation gives the evolution of a one-molecule probability-density function associated with the state, i.e., its position and velocity, of a molecule. Therefore, the one-molecule probability-density function is a function of 6 independent variables, notably, 3 for position space and 3 for velocity space. The numerical approximation of such high-dimensional problem is a daunting challenge, and requires efficient numerical approximation techniques.

To develop efficient numerical approximation techniques for the Boltzmann equation it is convenient to start with a simplified model problem. In this presentation a one-dimensional prototype Boltzmann equation is presented that contains most of the important properties of its higher-dimensional counterpart. Furthermore, goal-adaptive finite-element approximations of the prototype Boltzmann-equation are shown. Such approximations are optimal in the sense that they yield small errors in predefined goal-quantities (or macroscale quantities) compared to the required number of degrees-of-freedom. The results show that goal-adaptive strategies can be an important step forward in the approximation of the high-dimensional Boltzmann equation.