Kinetic transport in crystals and quasicrystals

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Abstract

The Lorentz gas is one of the simplest, most widely used models to study the transport properties of rarified gases in matter. It describes the dynamics of a cloud of non-interacting point particles in an infinite array of fixed spherical scatterers. More than one hundred years after its conception, it is still a major challange to understand the nature of the kinetic transport equation that governs the macrospopic particle dynamics in the limit of low scatterer density (the Boltzmann-Grad limit). Lorentz suggested that this equation should be the linear Boltzmann equation. This was confirmed in a series of celebrated papers by Gallavotti, Spohn, and by Boldrighini, Bunimovich and Sinai, under the assumption that the distribution of scatterers is sufficiently disordered. In the case of strongly correlated scatterer configurations (such as crystals or quasicrystals), we now understand why the linear Boltzmann equation fails and what to substitute it with. A particularly striking feature in these systems is a power-law tail for the distribution of free path lengths, and superdiffusive transport in the limit of large times.