An exact numerical solution strategy for regular hydrodynamical shock refraction at an inclined density discontinuity

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Introduction

We consider the problem of regular hydrodynamical refraction of a planar shock (S) at an inclined planar contact discontinuity (CD), seperating two gases at rest. When the shock impinges on the inclined density discontinuity, it refracts and 3 signals arise. Regular refraction means that these signals meet at a single point, called the triple point. After reflection from the top wall, the contact discontinuity becomes unstable due to local Kelvin-Helmholtz instability, causing it to roll up and form a Richtmyer-Meshkov instability (RMI). By solving for the conservative variables $\mathbf{u}(x,y,t)$ in the linear phase of the process, we can quantify the vorticity, ω_{CD}, deposited on the CD. An exact numerical solution strategy is presented, and compared to simulations performed by AMRVAC [2, 3]. We predict possible wave pattern transitions, which agree with experiments [1] and von Neumann theory [4]. We inverstigate the effect of a perpendicular magnetic field B.

Solution Strategy

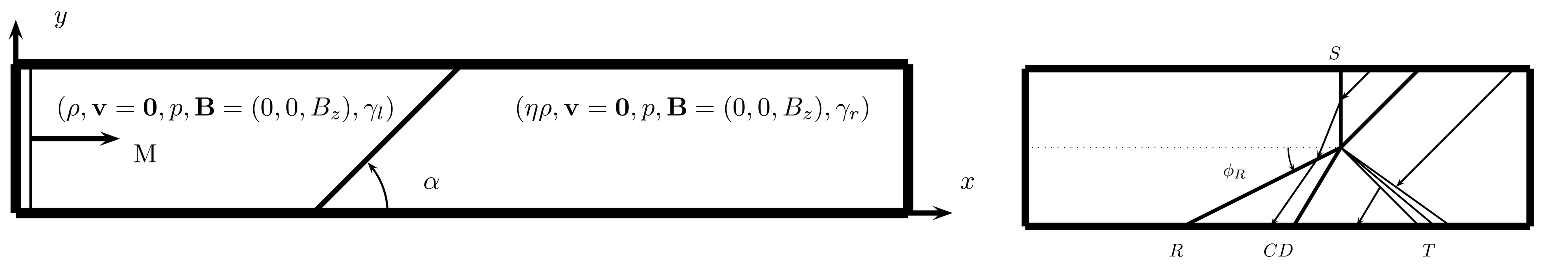


***Figure 1:** Left: Setup of the problem: a shock impinges on an inclined contact discontinuity. Right: The shock refracts in 3 signals. Both the refleted and transmitted signal can be a shock or an expansion fan. The total pressure p^* and the direction of the streamlines $\frac{v_y}{v_x}$ remain constant along the CD.*

- *Triple point moves aling CD*
- *Self-similar solution in frame of stationary triple point: $\mathbf{u} = \mathbf{u}(\phi)$*
- *Riemann problem around triple point*
- *Across expansion fans: numerical integration*
- *Across shocks: stationary Rankine-Hugoniot conditions*
- *p and $\frac{v_y}{v_x}$ are invariant across the CD*
- *initial guess for p^**
- *iteration on $[[\frac{v_y}{v_x}]](p^*)$*

Solution to the Riemann Problem

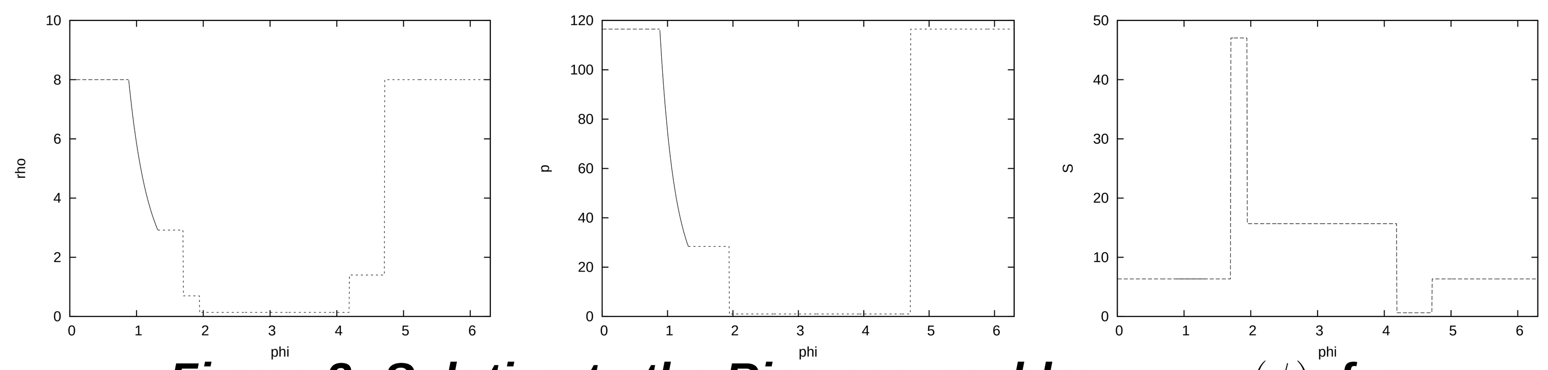


***Figure 2:** Solution to the Riemann problem $\mathbf{u} = \mathbf{u}(\phi)$, for $(\alpha, \beta^{-1}, \gamma_l, \gamma_r, \eta, M) = (\frac{\pi}{4}, 0, \frac{7}{5}, \frac{7}{5}, 3, 2)$.*

Abd-El-Fattah experiment

- *1978: fast/slow HD shock tube experiment [1]*
- *Very weak shock ($M = 1.12$) refracts at a CO_2/CH_4 interface*
- *Varying incident angle α*
- *von Neumann theory [4] predicts $\alpha_{crit} = 0.97$ and $\alpha_{trans} = 1.01$*

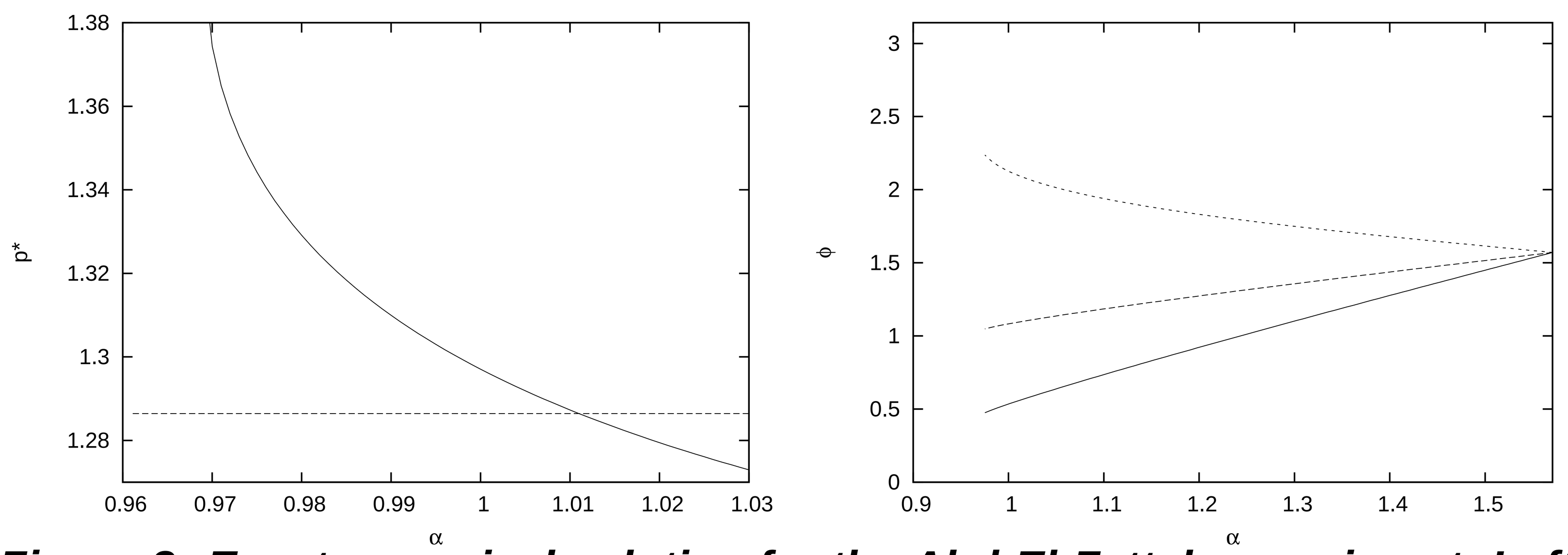


***Figure 3:** Exact numerical solution for the Abd-El-Fattah experiment. Left: $\alpha_{crit=0.97}$ and $\alpha_{trans} = 1.01$. Right: $\phi(\alpha)$.*

Connecting slow/fast and fast/slow refraction

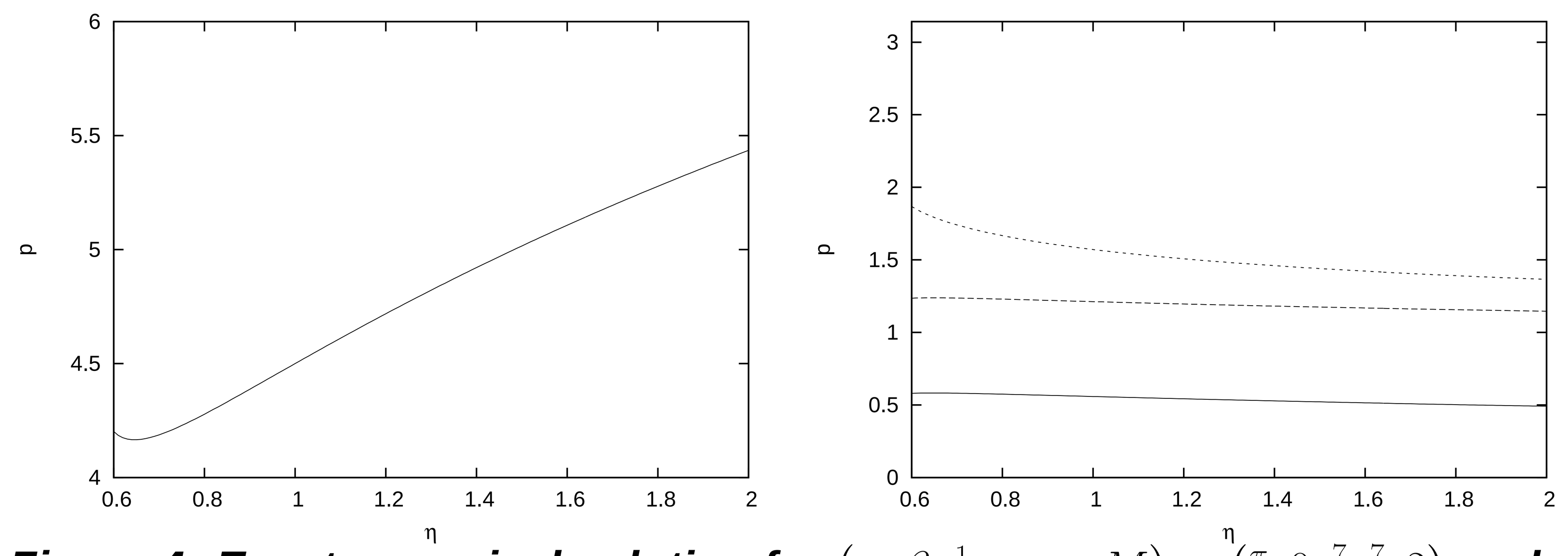


***Figure 4:** Exact numerical solution for $(\alpha, \beta^{-1}, \gamma_l, \gamma_r, M) = (\frac{\pi}{4}, 0, \frac{7}{5}, \frac{7}{5}, 2)$ and a varying range of η. Left: for $\eta < 1$ we have $p^* < p_{post} = 4.5$ and thus a reflected expansion fan, for $\eta > 1$ we have $p^* > p_{post} = 4.5$ and thus a reflected shock. Right: for $\eta < 1$: $\phi_T < \frac{\pi}{2}$ and for $\eta > 1$: $\phi_T > \frac{\pi}{2}$.*

Effect of perpendicular B

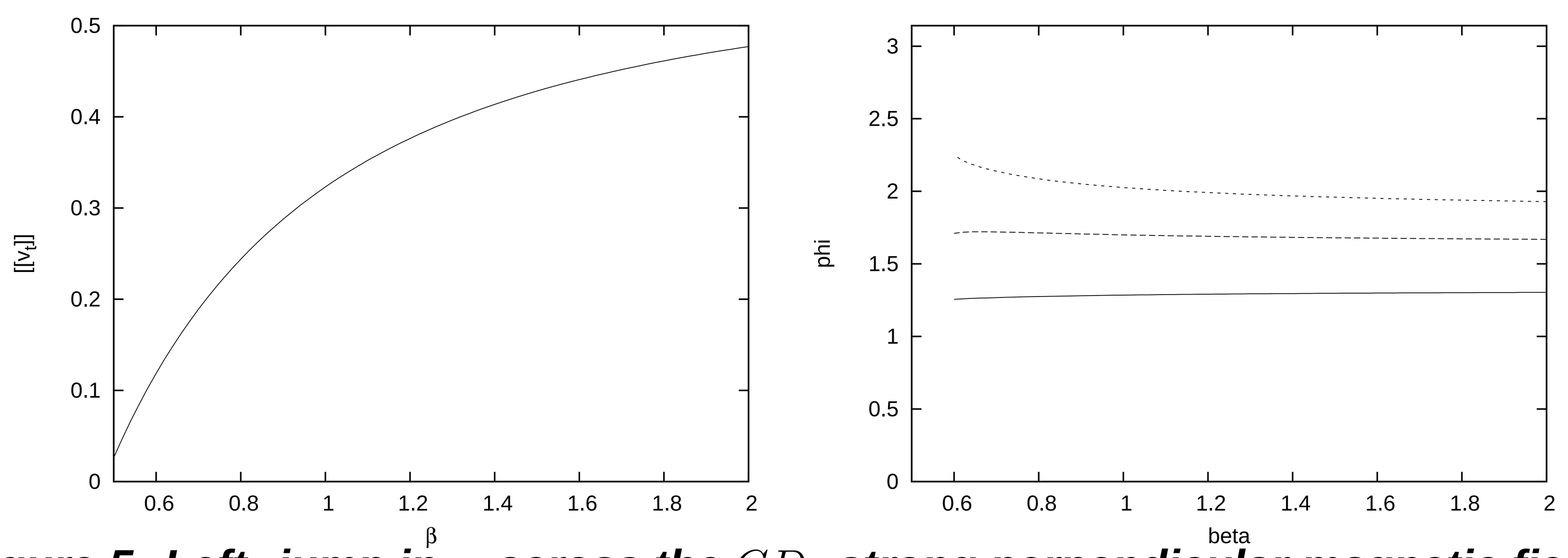
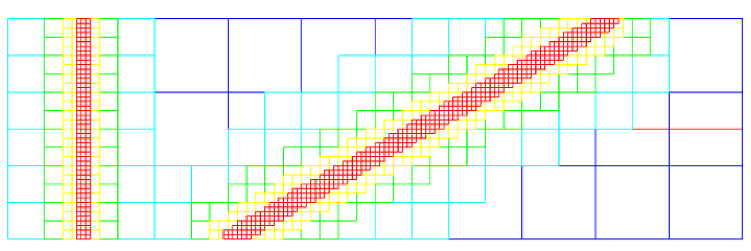


***Figure 5:** Left: jump in v_t across the CD: strong perpendicular magnetic fields suppress the instability of the CD. Right: Strong perpendicular magnetic fields slightly broaden the angles of the wave configuration.*

AMRVAC simulations

- *Adaptive mesh refinement*
- *Hybrid block-based*
- *MPI support*
- *Up to $3D$ relativistic MHD*
- *Solving equations of fluid dynamics: $\partial_t \mathbf{U} + \nabla \cdot \mathbf{F}(\mathbf{U}) = \mathbf{S}(\mathbf{U}, \mathbf{x}, t)$ and $\nabla \cdot \mathbf{B} = \mathbf{0}$*
- *Following an interface: $\partial_t(\rho\gamma) + \nabla \cdot (\rho\gamma\mathbf{v})$*



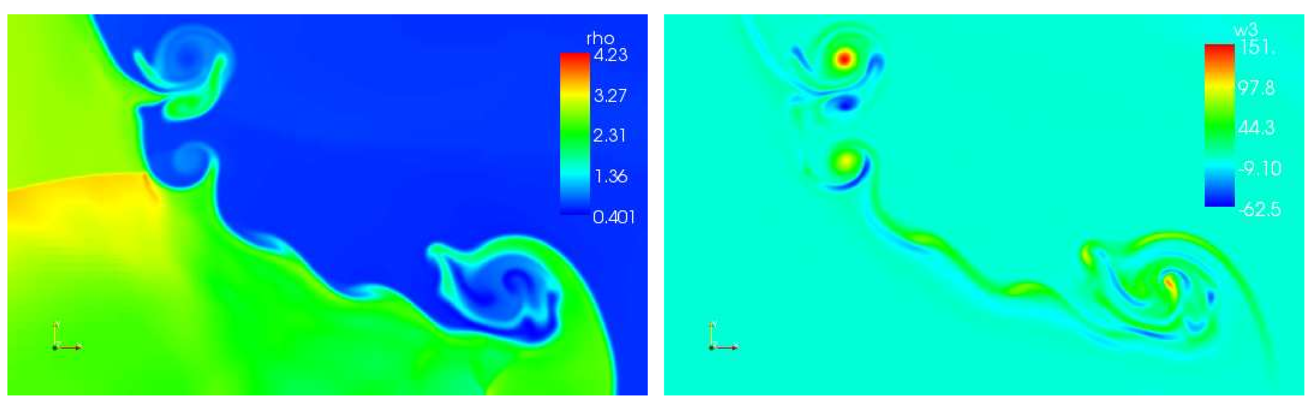


***Figure 6:** Simulation for $(M, \alpha, \beta, \gamma_l, \gamma_r, \eta) = \left(10, \frac{\pi}{4}, \frac{2}{9}, \frac{7}{5}, \frac{7}{5}, \frac{1}{10}\right)$: Left: Density plot; Right: Vorticity plot*

Conclusion

We developed an exact Riemann solver-based solution strategy for regular hydrodynamical shock refraction. Our results fit with numerical simulations and experiments. We will generalise our strategy for arbitrary B, where 7 signals arise.

References

[1] A.M. Abd-El-Fattah and L.F. Henderson, 1978, Shock waves at a fast-slow gas interface, J. Fluid Mech. **89**, 79-95.

[2] B. van der Holst and R. Keppens, 2007, Hybrid block-AMR in cartesian and curvi-linear coordinates: MHD applications, J. Comp. Phys. **226**, 925-946.

[3] R. Keppens, M. Nool, G. Tóth and J.P. Goedbloed, 2003, Adaptive Mesh Refinement for conservative systems: multi-dimensional efficiency evaluation, Comp. Phys. Comm. **153**, 317-339.

[4] J. von Neumann, 1963, Theory of games, astrophysics, hydrodynamics, and meteorology, In Collected Works, vol. 6.

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