

# Modeling sun shape and optical error in a solar tower power plant

## 1 Abstract

In this project, an existing simulation model designed to evaluate the efficiency of a solar tower power plant was modified and tested. The modification was done to take into account sun shape and optical error, two factors affecting the efficiency of this type of power plant. To estimate the effects of these factors, two different methods were developed and one of them was implemented in the existing model. Testing of the model was then done by comparing the total power output calculated by the model to the value calculated by Monte Carlo ray tracing software. The results of testing showed relatively close agreement between the Monte Carlo ray tracing and the simulation model.

## 2 Introduction

A solar tower power plant is made up of a large array of mirrors which are arranged to reflect sunlight towards the top of a tower. These large mirrors are called heliostats. At the top of the tower is a receiver, where some substance such as water or molten salt which is heated by the sunlight being reflected from the heliostats. Once heated, the thermal energy in this substance can be used to produce steam, which can then be used to power a generator and produce electrical energy for consumer use.

When designing this type of power plant, the most efficient layout is one that maximizes energy output and minimizes land usage. To achieve maximum efficiency, several parameters of the system can be varied, including the positions of the heliostats and the receiver. The simulation model used in this project evaluates the efficiency of any given arrangement of heliostats and receiver so that the efficiencies of various layouts can be compared. When evaluating a given arrangement, the model considers several factors which can reduce the efficiency of the power plant. In this project, the simulation model was improved and tested.

## 3 Project Description

### 3.1 Existing Simulation Model

The existing solar tower power plant simulation model was developed by Matthias Ewert and Omnieldis Navarro Fuentes. It estimates the efficiency of a solar tower power plant at any location on Earth at a specified date and time. The model first determines the sun position at the given location and time using an algorithm created by the Plataforma Solar de Almeria (PSA) [1]. Then it finds the average solar radiation using the Meteorological Radiation Model (MRM) [2]. The model then discretizes each heliostat into a two dimensional matrix of elements with each discrete element being represented by a single ray of light.

The behavior of each of these rays of light is then calculated, taking into account several factors which reduce the total power reaching the receiver. Atmospheric attenua-

tion causes the power of a ray to decrease as it moves through air and is modeled in this simulation by a mathematical function [5]. Cosine losses occur when a heliostat is not aligned perpendicular to the sun’s incoming rays, reducing the area of the heliostat that is seen by the sun and therefore the amount of light which is incident on each heliostat. Shading losses occur when a heliostat or the tower prevents sunlight from reaching a second heliostat. Similarly, blocking losses result when rays that are reflected from one heliostat do not hit the receiver because they hit the back of another heliostat. The existing simulation model takes all of these losses into account, as well as the reflectivity of the heliostat surface [3].

### 3.2 Improvement of the Model

In this project, two more important factors affecting the efficiency of a solar tower power plant were added to the existing model. The first was sun shape, which refers to the fact that the sun is not a point source and does not emit parallel rays of light [4]. The sun has a finite size when viewed from Earth and a ray coming from the sun could come from one of many locations in and around the sun’s area. This means that when a ray hits a heliostat at a certain point, it could make a number of different angles with the surface normal, producing a different angle of reflection than if the sun’s rays were parallel. In this model, the sun shape was modeled by a Gaussian function describing the probability that a ray will come from a certain location on the sun and therefore make a certain angle with the heliostat surface normal.

The second factor that was considered in this project was optical error [4]. This can be divided into two parts, slope and specular error. Slope error is associated with the macroscopic shape of the heliostat and occurs if the heliostat is not perfectly flat, but slightly warped. Specular error is related to the microscopic roughness of the heliostat. For both types of optical error, the irregular surface can cause a ray reflecting off a heliostat to project in a direction other than that predicted for an ideal flat surface. In our simulation model, a Gaussian distribution was again used, this time describing the probability that a reflected ray has a certain angle of deviation from the ideally predicted ray.

The improvement to the simulation model performed here took both sun shape and optical error into account using a single method. The two sources of error were first combined using the following equation:

$$\sigma_{\text{total}} = \sqrt{\sigma_{\text{slope}}^2 + \sigma_{\text{specular}}^2 + \sigma_{\text{sun}}^2}$$

The  $\sigma_{\text{total}}$  was then used to define a single Gaussian distribution describing the probability that a reflected ray leaves a heliostat with a certain angle of deviation from the predicted ray.

## 4 Implementing the Model Improvements

With a Gaussian distribution around the ideal reflected ray established, one can project the distribution function onto the receiver. This projection gives a distorted Gaussian distribution of a certain size and shape, depending on the distance the reflected ray travels and the angle it makes with the receiver. Integrating this projected distribution function

over the receiver area gives the probability that the real reflected ray is incident upon the receiver. The integration of this function on the receiver was attempted using two different methods, as described below.

## 4.1 Geometrical Method

### 4.1.1 Projection onto the perpendicular plane

The geometrical method of integration depended on simple geometry and therefore first required a transformation of the receiver into a plane orthogonal to the ideal reflected ray. By performing this transformation, it was guaranteed that when the Gaussian distribution was projected onto this new transformed plane, it would remain radially symmetric.

### 4.1.2 Defining the maximum radius cone

After transforming the receiver, a “maximum radius cone” centered around the ideal reflected ray was defined. This cone corresponded to a constant angle of deviation from the ideal reflected ray of three times the standard deviation,  $\sigma_{\text{total}}$ . When this cone was projected onto the transformed receiver, we obtained a “maximum radius circle.” If the circle was found to be entirely inside the receiver, then the probability of the reflected ray hitting the receiver was taken to be one. If it did not fit inside the receiver, the distribution function inside the “maximum radius circle” was broken up into parts, estimated and then summed.

### 4.1.3 Estimation of the integration by breaking into parts

If it was necessary to break the Gaussian distribution inside the “maximum radius circle” into parts for the integration, the first step was to find the largest circle that did fit inside the receiver, see figure 1. The angle defining a cone corresponding to a circle of that size was then determined by trigonometry. This angle was then divided by  $\sigma_{\text{total}}$  to calculate the number of standard deviations the reflected ray was from the ideal ray. The error function of this number, divided by a scaling factor of  $\sqrt{2}$ , was then determined. Since the error function is simply the integration of a Gaussian function, this evaluation gave the probability that the reflected ray would land inside the small circle.

The approximation of the integration was further improved by defining another circle that extended to the second closest receiver edge. The two circles created a ring around the inner circle. The probability that the reflected ray would fall inside this ring was calculated by evaluating the error functions at the two boundaries as described above and then taking the difference of these values. Using geometry, the fraction of this ring that was inside the receiver was estimated and multiplied by this probability to give a final probability associated with the second part of the integration estimation.

If the “maximum radius circle” happened to be smaller than the second circle, the method as above was used, but with the “maximum radius circle” being used in place of the second circle. If the “maximum radius circle” was larger than the second circle, the integration estimation continued with the calculation of a third and possible fourth part. The method of probability calculation used for the third and fourth parts was similar to that used for the second part. Again, if the “maximum radius circle” was reached before one of the other circles, the “maximum radius circle” was used for the outer boundary and no subsequent parts were calculated. Once the probabilities of each of the necessary

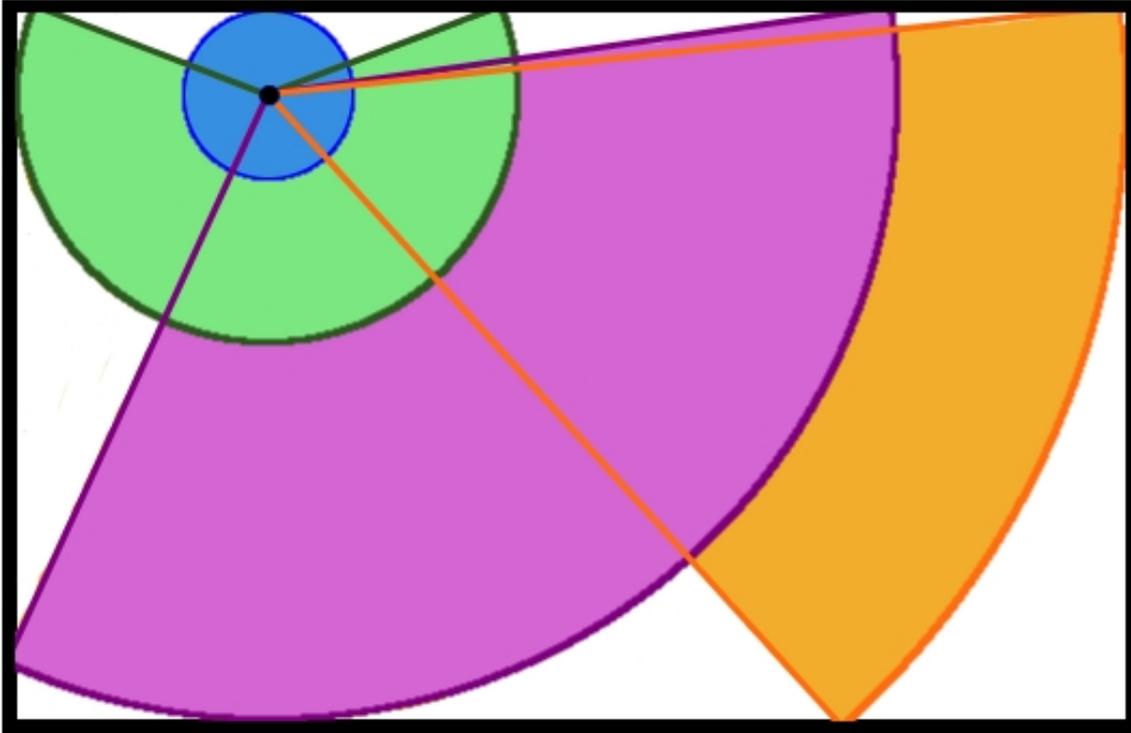


Figure 1: Schematic of the geometrical integration method.

parts were calculated, they were summed. Finally, the sum was normalized by dividing it by the probability of a ray landing within the “maximum radius circle.” The final calculated value was an estimation of the probability that the reflected ray would be incident upon the receiver.

## 4.2 Numerical Method

### 4.2.1 Gaussian Quadrature

The second method of integration that was attempted during this project was numerical integration by Gaussian Quadrature. In Gaussian Quadrature, a set of equations known as the Gauss-Legendre equations are used to define a set of points in a rectangular field. The function to be integrated is then simply evaluated at each of these points. The integration is the sum of these evaluations, each multiplied by appropriate weighting factors which are also determined by the Gauss-Legendre equations.

### 4.2.2 Mapping the Gaussian distribution on the receiver

When using Gaussian Quadrature, the integration must be performed over a rectangle, so the receiver cannot be translated into the perpendicular plane as before, since that could result in a non-rectangular four sided polygon. Therefore the integration must occur on the original rectangular receiver. To accomplish this, the Gaussian distribution must be projected into its distorted form on the receiver. This projection, which involves complex mathematics, is still in the process of being calculated and implemented in the simulation model. When the translation is completed, a simple evaluation of the distorted Gaussian distribution function at the prescribed points and multiplication by weighting factors

will give the integration of the function over the receiver. Again, this integration will represent the probability that the reflected ray is incident upon the receiver.

## 5 Testing the Model

### 5.1 Verification using SolTrace

The simulation model discussed here was verified by comparing it to a Monte Carlo ray tracing software package called SolTrace [6]. SolTrace works by sending hundreds of thousands of rays at the heliostats and tracing their paths after reflection. The software is capable of incorporating user-defined sun shape and optical error and can calculate the amount of power incident upon the receiver. The advantage of SolTrace is that the Monte Carlo method gives reliable results when enough rays are used. The disadvantage is that it is computationally expensive.

### 5.2 Verification Results

Verification of the improved model was performed for the case when the geometrical error integration method was used. The power incident upon the receiver was calculated using both the model and SolTRACE at thirty minute increments throughout a single day, see figure 2. The results showed relatively close agreement between the model and SolTRACE throughout the day. There is some discrepancy between the data sets and while we expect the geometric method to underestimate power, in fact the simulation model overestimates. This suggests that there could be some type of error in the model or that further refinements of the model are required to accurately estimate the efficiency of a solar tower power plant.

## 6 Evaluation

### 6.1 Performance of the Error Cone Method

When verifying the simulation model using SolTRACE, all of the various parts of implementation and calculation were being tested at once. Because of this fact, it was not possible to directly verify whether or not the Gaussian error estimation method was working as expected. Tests were done to compare the model with SolTRACE both with and without sun shape and optical error estimation enabled. Both cases showed discrepancies, suggesting that there could be errors in some other part of the model. Because of this fact, the method of sun shape and error approximation cannot be fully verified until other errors in the model are fixed.

### 6.2 Comparison of Error Calculation Methods

Although the method of numerical integration was not fully implemented, a comparison between the two methods would be a valuable analysis in the future. Once the numerical integration method is completed, the two could be compared to see how closely they agree with SolTRACE results and with each other. While the numerical integration is expected to be more accurate, it is also much more computationally expensive. Therefore, if the

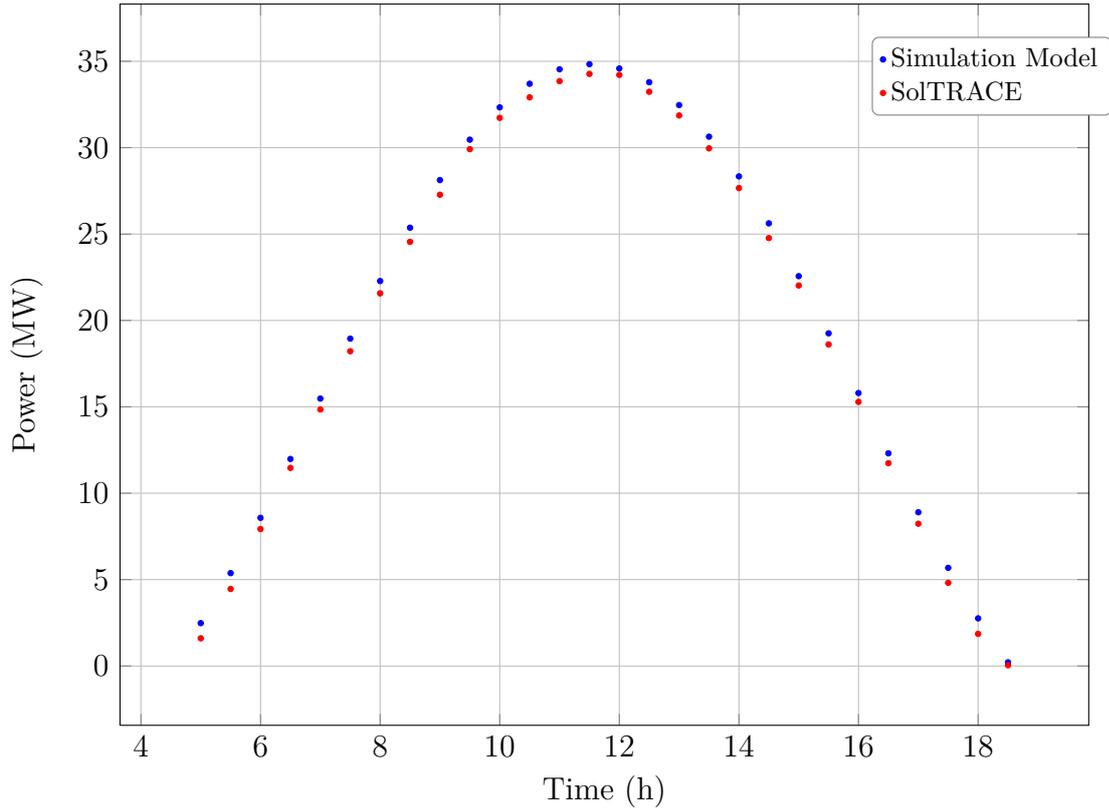


Figure 2: Verification of the model using SolTRACE

geometrical method gives results that are reasonably close to the numerical method, it may be beneficial to use the geometric method in the model to save computational time.

Another possibility that could be investigated is the combination of the two integration methods into a hybrid method. This might involve integrating some areas of the receiver geometrically and others numerically. It could also involve one method being used for certain cases, such as when the reflected ray from the heliostat reaches the receiver at a very sharp angle, and the other method being used in all other cases. The optimization of a hybrid integration method could be done in order to maximize accuracy while also minimizing computational time required.

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