Heliostat Field Optimization: A New Computationally Efficient Model and Biomimetic Layout

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Abstract

In this article, a new model and a biomimetic pattern for heliostat field layout optimization are introduced. The model, described and validated herein, includes a detailed calculation of the annual average optical efficiency accounting for cosine losses, shading & blocking, aberration and atmospheric attenuation. The model is based on a discretization of the heliostats and can be viewed as ray tracing with a carefully selected distribution of rays. The prototype implementation is sufficiently fast to allow for field optimization. Parameters are introduced for the radially staggered layout and are optimized with the objective of maximizing the annual insolation weighted heliostat field efficiency. In addition, inspired by the spirals of the phyllotaxis disc pattern, a new biomimetic placement heuristic is described and evaluated, which generates layouts of both higher insolation-weighted efficiency and higher ground coverage than radially staggered designs. Specifically, this new heuristic is shown to improve the existing PS10 field by 0.36 percentage points in efficiency while simultaneously reducing the land area by 15.8%. Moreover, the new pattern achieves a better trade-off between land area usage and efficiency, i.e., it can reduce the area requirement significantly for any desired efficiency. Finally, the improvement in area becomes more pronounced with an increased number of heliostats, when maximal efficiency is the objective. While minimizing the levelized cost of energy (LCOE) is typically a more practical objective, results of the case study presented show that it is possible to both reduce the land area (i.e. footprint) of the plant and number of heliostats for fixed energy collected. By reducing the capital cost of the plant at no additional costs, the effect is a reduction in LCOE.

Keywords

concentrated solar; central receiver; heliostat field; biomimetic; phyllotaxis

1 Introduction

A new model is presented for the calculation of heliostat field optical efficiency, accounting for all significant factors affecting the performance of central receiver solar thermal systems including, (i) cosine losses, (ii)
shading and blocking, (iii) receiver interception (i.e., heat not lost due to spillage), (iv) atmospheric attenuation between heliostat and receiver, and (v) heliostat reflectivity. The definition of each of these terms is common in the open literature [Schmitz et al.(2006)] and a detailed description of their calculation in the model is presented in Section 2 (except for heliostat reflectivity which is assumed constant).

The purpose of developing this model, written in object-oriented Fortran 95, is for the gradient-based optimization of both traditional and non-traditional heliostat field layouts (e.g., on hillsides [Noone et al.(2011), Slocum et al.(2011)]). This results in two main requirements, namely (i) computationally efficient calculation of efficiency with high accuracy and (ii) suitability for differentiation using algorithmic differentiation (AD) tools [Griewank and Walther(2008), Hascoët and Pascual(2004)]. This article describes the new model and proposes an improved biomimetic pattern for heliostat placement which substantially improves on the existing heuristics.

As a result of the intended purpose of optimization of heliostat field layouts, the proposed model was developed considering accuracy and computational efficiency, allowing for several distinguishing approaches to implementation. Firstly, Monte Carlo ray-tracing tools, such as SolTRACE [Wendelin(2003)], are accurate with a sufficient number of cast rays; however, the drawback is that they are computationally expensive for the purpose of evaluating instantaneous optical efficiency of large heliostat fields and are not a practical option for optimization of annual optical efficiency. SolTRACE is however used in Section 3 to validate the proposed model in small tests involving only the instantaneous heliostat field efficiency and the results of the two models show excellent agreement.

Secondly, the most expensive efficiency evaluation, that of shading and blocking, is calculated using a discretization of the heliostat surface. With a relatively coarse discretization, this method is both faster than calculating the intersection exactly and is sufficiently accurate, i.e., the error incurred by this method is much smaller than the variation in optical efficiencies of the fields presented, as determined by a mesh refinement study and discussed in Section 3. Other approximation methods in the open literature assume that shading and blocking effects between multiple heliostats are distinct and therefore can be treated pairwise [Kistler(1986)] (i.e., the shaded or blocked region of a heliostat is neither shaded nor blocked by another heliostat), or in an even more simplified case, that shading may be neglected and blocking assumed to be constant [Collado and Turegano(1989)]. However, with a hierarchical approach [Belhomme et al.(2009)] to evaluating shading and blocking, these assumptions are not necessary at the heliostat level because the source of shading or blocking does not matter at the discretization level. In other words, the proposed model is similar to ray-tracing techniques in that whether a region shaded by one heliostat is also blocked by another heliostat is of no consequence because the ray does not reach its intended target in any scenario. If shading and blocking is calculated between heliostats pairwise, the resulting efficiency is either a lower or upper bound depending on whether the modeler assumes the pairwise effects are distinct or completely overlapping, respectively.

Thirdly, the receiver flux calculation is more accurate than assuming a single error cone originating at the center of the heliostat, because the proposed model uses the surface normal at each discretization (or facet) to determine the direction of the reflected rays as a function of orientation on the heliostat. As a result, the effect of the time-variant optical error of aberration (i.e., astigmatism) is directly accounted for.

Finally, rather than iterating with a constant time step size, as is common practice in the open literature [Kistler(1986), Elsayed et al.(1992), Yao et al.(2009)], the model presented takes steps in the solar state space. In the middle of the day when insolation is at peak and the solar azimuth changes the fastest (with respect to time), the proposed method takes small time steps relative to sunrise and sunset. This approach allows the model to retain the same accuracy as a constant time step implementation but requires much fewer iterations, see Section 3.

The structure of the subsequent sections is as follows. Section 2 describes all major aspects of the model, namely the calculation of solar position and insolation as well as each of the aforementioned efficiency factors presented in Equation (1). Section 3 validates the most computationally extensive components of the model (and thus most error-prone), specifically shading and blocking efficiency and interception efficiency. In terms of both implementation and computational expense, the much simpler evaluation of cosine, atmospheric attenuation, and reflectivity efficiencies are discussed in Section 2 and the implementations are validated.
with the open literature but do not warrant a detailed discussion. Section 4 demonstrates the application of the model by evaluating the design of the commercial scale power plant, PS10, as well as a new radially staggered configuration which performs better. Section 5 presents a new biomimetic spiral heuristic and is shown to outperform even the optimized radially staggered configuration. Lastly, Section 6 summarizes the model, applications, and heliostat placement heuristics presented.

2 Model Description

The instantaneous efficiency is calculated as the product of the instantaneous efficiency terms introduced in Section 1, where $\eta_{\text{cos}}$ represents cosine losses, $\eta_{\text{sb}}$ is shading and blocking, $\eta_{\text{itc}}$ is the interception of sun rays at the aperture, $\eta_{\text{aa}}$ is the atmospheric attenuation, and $\eta_{\text{ref}}$ is the heliostat reflectivity.

$$\eta = \eta_{\text{cos}} \cdot \eta_{\text{sb}} \cdot \eta_{\text{itc}} \cdot \eta_{\text{aa}} \cdot \eta_{\text{ref}}.$$  \hfill (1)

Additionally, two average annual heliostat field efficiencies are calculated, the unweighted $\eta_{\text{year},\text{i}}$ and the insolation weighted $\eta_{\text{year},\text{i}}$. The equation for the unweighted efficiency is identical to the insolation weighted efficiency with the exception that the insolation term $I_b(t)$ is removed.

$$\eta_{\text{year},\text{i}} = \frac{\sum_{\text{day}=1}^{365} \int_{\text{sunrise}}^{\text{sunset}} I_b(t) \eta(t) dt}{\sum_{\text{day}=1}^{365} \int_{\text{sunrise}}^{\text{sunset}} I_b(t) dt}. \quad (2)$$

2.1 Solar Position

In order to calculate the instantaneous heliostat field efficiency, as shown in Equation (1), a model for solar position is implemented. The solar position computational model [Duffie and Beckman(2006)] requires a negligible computational expense compared to the evaluation of the efficiency terms, is sufficiently accurate (i.e., the errors do not significantly affect the optimized layout), and has a simple functional form which is beneficial in the implementation of automatic differentiation tools used with gradient-based optimization algorithms. The solar declination, $\delta$, and hour angle of sunrise and sunset as a function of day number, $n_d$, and latitude, $\phi$ (all angles in radians), are calculated as

$$\delta = \frac{23.45 \pi}{180} \sin \left( \frac{2 \pi}{365} \cdot \frac{284 + n_d}{365} \right) \quad (3)$$

$$\omega_{\text{sunrise}} = \cos^{-1} (\tan \phi \tan \delta) - \pi = -\omega_{\text{sunset}} \quad (4)$$

The sun’s position relative to an observer on the ground is described by two angles, the solar altitude and azimuth [Noone et al.(2011)]

$$\alpha = \sin^{-1} (\cos \phi \cos \delta \cos \omega + \sin \phi \sin \delta) \quad (5)$$

$$\gamma = \text{sgn}(\omega) \cos^{-1} \left( \frac{\sin \alpha \sin \phi - \sin \delta}{\cos \alpha \cos \phi} \right) \quad (6)$$

While these equations are sufficiently accurate for heliostat placement, if more accurate solar positioning is required, models such as the Solar Position Algorithm (SPA) [Reda and Andreas(2008)] are available.
2.2 Insolation

The model used for estimating hourly solar radiation is the first version of the Meteorological Radiation Model (MRMv1) [Badescu(2008)], which accounts for several transmittance terms, but assumes cloudless skies. While this assumption prohibits the use of the model in much of the world, sites which are suitable for CSP rarely have extended periods of cloudy, overcast, or hazy weather. This model has been validated with measurements from select locations in the United States and southern European countries and has shown acceptable accuracy in capturing hourly, daily, and seasonal variations. While the MRM is used herein, countless alternate radiation models in the literature, and even measured data, can be used instead; insolation serves primarily as a weighting function.

2.3 Cosine Efficiency

The calculation of cosine efficiency is extremely simple using the Law of (specular) Reflection. The dot product of the directions of sun and heliostat (or facet) normal direction is related to the angle of incidence, \( \theta_i \).

\[
\eta_{\text{cos}} = \cos \theta_i = \hat{d}_{\text{sun}} \cdot \hat{d}_n \tag{7}
\]

For a paraboloid, the surface normal direction is calculated at any position by differentiating the equation of the surface, where \( z \) is the normal direction of the center of the heliostat in the local coordinate system and \( f \) is the focal distance.

\[
z - \frac{x^2 + y^2}{4f} = 0 \tag{8}
\]

\[
\hat{d}_n = \frac{1}{\sqrt{x^2 + y^2 + 4f^2}} \begin{bmatrix} -x \\ -y \\ 2f \end{bmatrix} \approx \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \quad \text{(for } f \gg x, y) \tag{9}
\]

2.4 Shading and Blocking Efficiency

An exact calculation of shading and blocking for a single heliostat, \( h_i \), is accomplished by first calculating the pairwise intersection of \( h_i \) with every other heliostat in the directions of the sun and receiver, then finding the union of these intersections. Due to the computational expense, few codes, e.g., [Lipps and Vant-Hull(1980)], for heliostat field optimization calculate shading and blocking exactly; instead, approximation methods are commonly used to eliminate the step of computing the union of shaded and blocked regions. Some commercial codes, e.g., TieSOL, may have exact calculation as well.

An extremely accurate, simple to implement, and computationally inexpensive alternative is to use a discretization of \( h_i \), as shown in Figure 1(a). The discretization points are projected in the directions of the sun and receiver for shading and blocking, respectively. If the projected point intersects the interior of another heliostat, the representative area on \( h_i \) is either shaded or blocked, while a point on the exterior is neither. For the purpose of increased accuracy (relative to only calculating whether or not intersection occurs), if the point is near the heliostat edge, a region of width equal to the discretization distance (\( \delta s \) as shown in Figure 1(b)), the amount of shading and blocking is interpolated as a function of distance from the edge. The interpolation method chosen can also be used to ensure that the model is differentiable, which may also be satisfied by time integration. The reason shading and blocking are combined into a single term is that in the implementation of the model, the shading and blocking efficiency of each discretization point is calculated as the minimum ratio of useful area to total area after projecting to each of the other heliostats. For example, if the area represented by discretization point \( p \) is partially shaded by \( h_i \) and partially blocked by \( h_j \), the interaction which shades or blocks the largest area represents the efficiency of the discretized area. The shading and blocking efficiency of the entire heliostat is then calculated as the sum of the area neither shaded nor blocked in each discretization divided by the total heliostat area. In the prototype implementation
used herein, the discretization points are calculated such that the separation distance in both the heliostat width and height is uniform. Recognizing that the effect of shading and blocking is typically small and thus confined to the edges, it would be advantageous to use a fine discretization near the heliostat edge and a coarse discretization in the middle; however, for the purpose of this article, only a uniform discretization is considered with six points on each side.

Figure 1: Schematic of Discretized Heliostat (a) and Calculation of Shading and Blocking of an Individual Discretization Point Projected to Another Potentially Shading or Blocking Heliostat (b)

The computational complexity of a pairwise comparison of heliostats can be reduced by only considering a subset of heliostats that can potentially shade or block heliostat \( h_i \). This method, also referred to as the bounding sphere method [Belhomme et al.(2009)], prevents unnecessary calculations for heliostats that are incapable of shading or blocking the heliostat currently being evaluated. Figure 2 illustrates how the method is used to determine whether \( h_2 \) is included in the subset of heliostats capable of potentially blocking heliostat \( h_1 \). If the distance \( d \) between \( h_2 \) and the line segment connecting \( h_1 \) and the receiver (or the ray from \( h_1 \) in the direction of the sun) is less than the sum of the radii of the bounding spheres, \( r \), then \( h_2 \) potentially blocks (or shades) \( h_1 \). By maintaining two lists of potentially shading and blocking heliostats for each \( h_i \), the complexity of the pairwise intersection calculation is reduced from \( O(n^2) \) to \( O(nm) \), where \( n \) is the total number of heliostats and \( m \) is the subset of heliostats evaluated for shading and blocking (typically \( m \ll n \)). Regardless of how shading and blocking is implemented, the much simpler calculation of the distance between a point and line segment significantly reduces the instances of the much more expensive calculation of shading and blocking. Additionally, while the subset of potentially shading heliostats needs to be updated for each time step due to the motion of the sun, the centers of the heliostats and receiver aperture are fixed with respect to time, so the subset of potentially blocking heliostats needs to be calculated only once.

2.5 Interception Efficiency

The interception efficiency is calculated as a result of factors including off-axis aberration (i.e., astigmatism), surface errors such as microscopic imperfections and slope errors, tracking errors, and sun shape. The proposed model uses a similar discretization method as in the calculation of shading and blocking to determine the direction of the reflected rays at each point and the intersection of the reflected error cone with the plane of the aperture. The direction of the reflected ray as a function of direction of the sun (from the point of view of an observer) and the surface normal direction of the discretization (or facet) is calculated as

\[
\hat{d}_{\text{ref}} = 2 \left( \hat{d}_n \cdot \hat{d}_{\text{sun}} \right) \hat{d}_n - \hat{d}_{\text{sun}}
\]
Similar to existing codes [Schmitz et al.(2006), Garcia et al.(2008), Collado(2008), Yao et al.(2009)], the error cone of the reflected ray is approximated with flux density proportional to an angular Gaussian distribution [Rabl(1985)]. To do so, the ray corresponding with the heliostat surface discretization is intersected with the plane of the receiver and the inverse error function is used to approximate the integration of the error cone over the surface of the receiver as a function of minimum distance to the receiver edge. This one-dimensional approximation provides sufficiently accurate results compared to the more expensive ray tracing, since the flux distribution is small relative to the receiver dimensions. The interception efficiency is then calculated as the integral of the power incident to the aperture divided by the total power incident to the plane of the aperture. As described in the following, the power incident to the plane of the aperture is the specular reflectance from the heliostats minus the atmospheric attenuation between heliostat and receiver. It is assumed in the model that each heliostat aims toward the center of the aperture. In reality, complex time-variant targeting strategies reduce the flux at the center of the aperture by (intentionally) spreading the incident radiation across the surface of the aperture. While these strategies mitigate the risk of dangerous flux levels at the center, the result is a lower interception efficiency. Therefore, the interception efficiency presented herein is an upper bound.

An advantage of using a discretization of the heliostat is that the curvature of the heliostat is accounted for when calculating the normal direction of each point during initialization of the heliostat field. Standard “on-axis” focusing or canting is modeled by fixing the focal point along the optical axis at a defined distance. The same procedure is used for asymmetric curvatures by defining a direction or solar position in which each heliostat is designed to focus. After the normal directions of the discretization points are defined, they too are rotated along with the position of the heliostat at each time step.

2.6 Atmospheric Attenuation Efficiency

Atmospheric attenuation accounts for radiation losses in the distance $d_{rec}$ between a heliostat and the receiver and is calculated as

$$\eta_{aa} = \begin{cases} 
0.99321 - 0.0001176d_{rec} + 1.97 \cdot 10^{-8}d_{rec}^2 & d_{rec} \leq 1000m \\
\exp(-0.0001106d_{rec}) & d_{rec} > 1000m 
\end{cases}$$

(11)

for $d_{rec}$ in meters [Schmitz et al.(2006)]. These losses are approximated assuming a visibility distance of about 40 km. Similar equations exist in the open literature for varying visibilities [Hottel(1976), Kistler(1986)], but the difference between models is less than 1% in the range of visibilities typical of a clear day. While other more detailed models exist, which account for site and weather specific inputs [Pitman and Vant-Hull(1984)], the impact of replacing the above model on the optimized heliostat layout would be small as the sensitivity to the optimization variables (i.e., heliostat spacing) is not as significant as cosine, shading and blocking, and interception efficiency. While only one site is considered in the case studies presented, a detailed model might be necessary if multiple sites are considered.
3 Model Validation

The most computationally extensive components of the model (and thus most error-prone), described in Section 2, specifically the terms shading and blocking efficiency and interception efficiency, are validated using the ray-tracing tool SolTRACE [Wendelin(2003)]. Additionally, the dependence of efficiency on discretization size and time step size are also investigated.

The model is validated with SolTRACE for the heliostat fields shown in Figures 6 and 7. Using a heliostat discretization of only 9 points (25 discretization points are shown in Figure 1(a)), the instantaneous error (i.e., single time step) in the calculation of shading and blocking efficiency is less than $10^{-3}$. While this error is sufficiently small for heliostat placement, the error reduces to less than $10^{-4}$ using a grid of 100 points. This error is even further reduced when averaged annually and is sufficiently small such that the improvements in efficiency presented in the remainder of the paper are the result of the proposed heliostat field layouts and not artifacts of the model.

<table>
<thead>
<tr>
<th>Table 1: Interception Efficiency Validation Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Location and Time</strong></td>
</tr>
<tr>
<td>Latitude</td>
</tr>
<tr>
<td>Day</td>
</tr>
<tr>
<td>Hour</td>
</tr>
<tr>
<td>Receiver Position (x,y,z) (0,0,115)m</td>
</tr>
<tr>
<td>Receiver Normal Direction (0,0.9763,−0.2165)</td>
</tr>
<tr>
<td>Heliostat Position (300,150,5)m</td>
</tr>
<tr>
<td>Heliostat Width 12.84m</td>
</tr>
<tr>
<td>Heliostat Height 9.45m</td>
</tr>
</tbody>
</table>

Interception efficiency was also validated using SolTRACE by first checking that the image of the concentrated light on the receiver (i.e., heliostat projection) is consistent between the two programs. Shown in Figure 3(a) is the image produced by a single heliostat at the configuration shown in Table 1 assuming an ideal paraboloidal geometry (i.e., no facets or optical errors) and a point light source. Figure 3(b) is the image produced by SolTRACE, which shows excellent agreement. Furthermore, an advantage of the implementation is that the time-dependent effects of off-axis aberration are directly accounted for by using a discretization of the heliostat.

Based on the location of the discretization points and the direction of the reflected rays, the intersection of the rays with the receiver is known (Figure 3(a)). Then, the calculation of interception efficiency simply combines the optical errors, sun size, and the finite size of the facets into a representative error cone and an integration of the intersection of the error cone with the surface of the receiver. The resulting flux map from Figure 3 is shown in Figure 4, which was validated in SolTRACE by modeling the individual heliostat facets. Again, after including optical errors, the spot size and intensity show excellent agreement.

As aforementioned, the time-averaging in the proposed model is performed with constant steps in the solar state space. This results in the same accuracy of the calculated insolation-weighted efficiency as a constant time step implementation with much fewer iterations. The validity of this claim is demonstrated in Figure 5 for the PS10 case study. Both methods require in the order of thousands of steps for a sufficiently accurate calculation. However, the proposed method requires approximately half the number of steps for a given accuracy. In the following case studies time-averaging is performed using 3,684 steps.

4 PS10 Validation and Redesign

PS10, the 11MWe power tower plant located in Andalusia, Spain [Osuna et al.(2000)], is analyzed with the presented model for additional validation and benchmarking purposes. As detailed in Table 2, the heliostat
Figure 3: Validation of Heliostat Projection (neglecting optical errors) Shown in the Local Coordinates of the Receiver Aperture

Figure 4: Validation of Spot Size (including optical errors) using SolTRACE

field consists of 624 heliostats, each roughly 120m$^2$, arranged in a radially staggered configuration north of the 115m receiver tower. For the purposes of this analysis, the heliostat configuration is assumed to be planar despite the small variation in elevation of the PS10 site, with a maximum difference in elevation across the field of approximately 10 meters.

The first test case is the existing PS10 layout, generated by the Sandia code WinDELSOL1.0 [Kistler(1986), Wei et al.(2010)], shown in Figure 6(a). The resulting annual unweighted efficiency is 64%, the same value provided by Abengoa Solar as well as results from previous modeling efforts [Wei et al.(2010)]. Table 3 details a breakdown of the individual efficiency factors described in Section 2.

Without changing any aspect of the plant other than heliostat positions, the layout of PS10 was redesigned using the presented model with a similar radially staggered heliostat placement heuristic described in Section 4.1. The new layout, shown in Figure 6(b) increases the optical efficiency $\eta_{\text{year},I}$ by 0.19 percentage points and decreases the land area by about 10.9%, while maintaining the constraint that nearby heliostats must be separated by a minimum distance such that collision is not possible. Land area is defined as the area of the convex hull of the $(x, y)$ positions of the tower and heliostats. Table 3 lists the individual efficiency factors. While the effect of atmospheric attenuation is greater in the improved radially staggered layout due to an increase in average distance from heliostat to receiver, interception efficiency is slightly improved by
Figure 5: Comparison of Constant Time Step Averaging with Constant Solar Step Method.

placing the heliostats closer to the normal direction of the receiver aperture so the intersection of the error cone with the aperture produces a smaller spot size. Most notably however, is that by placing the heliostats closer together, the total area of the field can be reduced significantly without sacrificing efficiency simply by trading a lower shading and blocking efficiency for a higher cosine efficiency.

4.1 Methodology of Heliostat Field Optimization

The heliostat field layout in Figure 6(b) was chosen by optimizing two parameters of the pattern which control the increase in distance between successive rows and when the rows split (i.e., when the distance between heliostats in a row is large enough that the angular separation between heliostats in the successive row can be split in half without the potential for heliostat collision or causing significant shading or blocking). The nominal values for radial and azimuthal (angular) spacing are from the DELSOL user’s manual [Kistler(1986)], based on previous work [Walzel et al.(1977), Lipps and Vant-Hull(1978)], where $\theta_L$ is the receiver aperture altitude angle with respect to a position on the ground (function of row radius), HM is the heliostat height, WM is the heliostat width, and THT is the tower height. The radial spacing has been divided by two from the original form of the equation to represent distance between successive rows.
Table 2: PS10 Heliostat Field Parameters

<table>
<thead>
<tr>
<th>Location</th>
<th>Latitude 37°26’ N</th>
<th>Longitude 6°15’ W</th>
</tr>
</thead>
<tbody>
<tr>
<td>Heliostats (Sanlúcar 120)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Count</td>
<td>624</td>
<td></td>
</tr>
<tr>
<td>Width</td>
<td>12.84m</td>
<td></td>
</tr>
<tr>
<td>Height</td>
<td>9.45m</td>
<td></td>
</tr>
<tr>
<td>Reflectivity</td>
<td>0.88</td>
<td></td>
</tr>
<tr>
<td>$\sigma_{\text{optical}}$</td>
<td>2.9mrad</td>
<td></td>
</tr>
<tr>
<td>$\sigma_{\text{sun shape}}$</td>
<td>2.51mrad</td>
<td></td>
</tr>
<tr>
<td>Facet Canting</td>
<td>on-axis parabolic</td>
<td></td>
</tr>
</tbody>
</table>

| Receiver          | Tower Height 115m | Normal Direction (0, 0.9763, -0.2165) | Aperture Width 13.78m | Aperture Height 12m |

| Solar Model Parameters | | |
| Distribution of Aerosol Particles, $\alpha$ | 1.3 | |
| Turbidity Coefficient, $\beta$ | 0.1 | |

\[
\Delta r = \frac{1}{2} (1.14424 \cot \theta_L - 1.0935 + 3.0684 \theta_L - 1.1256 \theta_L^2) \cdot \text{HM} \tag{12}
\]

\[
\Delta az = \left(1.7491 + 0.6396 \theta_L + \frac{0.02873}{\theta_L - 0.04902}\right) \cdot \text{WM} \frac{2r}{2r - \text{HM} \cdot \Delta r} \left(1 - \frac{\text{HM} \cdot \Delta r}{2r \cdot \text{THT}}\right)^{-1}, \tag{13}
\]

where all the dimensions have to be taken in meters, since the equation is not dimensionally consistent.

In the implementation of the model, a multiplier is added to the $\Delta r$ term to modify the radial growth. Additionally, another parameter is used to control the azimuthal spacing (when the row splits) and is defined by the ratio of the angular separation of the current row and the $\Delta az$ term above. When this ratio exceeds the parameter value, the angular spacing of the row is halved from the previous row. This parameter optimization requires a computationally efficient model and is the primary reason why traditional ray-tracing tools were not used.

Specifically, the individual locations of the heliostats are determined by generating a pattern much larger than the expected size of the actual heliostat field using a known heuristic (e.g., the radially staggered pattern) and selecting parameter values (in this case, there are two parameters) which control how large the pattern is and/or how quickly it grows. In the case studies presented herein, the number of heliostats in the candidate field is taken as five times the number of desired heliostats. Next, all of the candidate locations are evaluated in the absence of shading and blocking in order to quickly calculate efficiency neglecting interaction between heliostats. These candidate locations are then sorted by their individual efficiencies and the $n$ heliostats (624 for the PS10 case study herein) with the highest efficiency are chosen to comprise the field. Note that the approach of selecting from a larger candidate field has been utilized in the literature, e.g., [Pitz-Paal et al. (2011)]. Finally, this field is then evaluated with all of the efficiency factors (including shading and blocking) to calculate the average annual insolation weighted efficiency of the entire field.

This process of generating the over-sized candidate set of heliostats, evaluating the set, and then selecting a prescribed number of heliostats is repeated for a finite set of parameter combinations in order to maximize the objective $\eta_{\text{year,1}}$ (as done for the improved radially staggered and phyllotaxis patterns, Figures 6(b) and 7(a), respectively). In the phyllotaxis pattern described in Section 5, the parameters optimized are $a$ and $b$.
Biomimetic Heuristic

For tower receivers, the efficiency of an individual heliostat is typically higher near the tower than far away, see also Figures 8(a) and 9(a). Therefore, it seems preferable to have a higher density of heliostats near the tower than far away even at the expense of increased shading and blocking. The disadvantage of the radially staggered configuration is that the transition from high to low density is not continuous (unlike efficiency). Therefore, a new heuristic is presented, inspired from spiral patterns of the phyllotaxis disc, which has the advantage of a continuous density function. An example of phyllotaxis is the configuration of florets on the head of a sunflower [Vogel(1979)], taking the form of Equations (14) and (15), where $\theta_k$ is linearly proportional to the $k^{th}$ element of the sequence and $r_k$ is the radial growth function, expressed by the constant exponentiation of $k^{th}$ element. The angular component is related to the golden ratio $\phi$, which equals $\frac{1+\sqrt{5}}{2}$. In the example of sunflowers, the coefficient $b$ in the radial component equals 0.5, resulting in the form of Equation (15) known as Fermat’s spiral. However, when $b$ is 0.5, the mean distance between neighboring florets in the sunflower model is constant [Jean(1994), Ridley(1982)]. In heliostat fields, it is beneficial to vary the heliostat pattern density as a function of distance from the receiver, which is accomplished by allowing $b$ to vary.

\begin{align*}
\theta_k &= 2\pi\phi^{-2k} \quad (14) \\
r_k &= a k^b \quad (15)
\end{align*}

Applying the spiral pattern to heliostat placement yields the result shown in Figure 7(a). The values of the coefficients $a$ and $b$ are 4.5 and 0.65, respectively, and are obtained with the same approach of parameter optimization as the radially staggered results presented in Section 4.1, optimizing the heliostat field layout for combinations of $a$ and $b$ in the ranges of $[2,8]$ and $[0.5,0.7]$, respectively. Figure 7(b) illustrates the Pareto curve as it represents the trade-offs between the size of the heliostat field and efficiency across the range of parameters selected for both the radially staggered and phyllotaxis spiral patterns. In addition to the parameters shown, the center of the spiral pattern was varied along the north-south direction with respect to the tower; however, optimal field configurations resulted when the spiral pattern is centered at the receiver tower. The value of $\phi$ is not optimized because even small variations from the nominal $\frac{1+\sqrt{5}}{2}$ produces dramatically different patterns which were significantly suboptimal. While other radial growth functions are considered, namely the logarithmic and Archimedean spirals, neither perform better than the growth

Figure 6: Comparison of the Original PS10 Configuration with the Redesigned Configuration
As detailed in Table 3, the spiral pattern significantly outperforms the radially staggered pattern. Compared to the design of PS10, the spiral pattern has an efficiency $\eta_{\text{year}, I}$ 0.36 percentage points higher while simultaneously reducing the size of the heliostat field by 15.8%.

Table 3: PS10: Breakdown of Heliostat Field Efficiency Terms and Heliostat Field Area

<table>
<thead>
<tr>
<th></th>
<th>radially staggered, WinDELSOL1.0</th>
<th>radially staggered, MIT</th>
<th>phyllotaxis spiral, MIT</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(Figure 6(a))</td>
<td>(Figure 6(b))</td>
<td>(Figure 7(a))</td>
</tr>
<tr>
<td>Unweighted Heliostat Field Efficiency</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\eta_{\text{cos}}$</td>
<td>0.8283</td>
<td>0.8308</td>
<td>0.8310</td>
</tr>
<tr>
<td>$\eta_{\text{sb}}$</td>
<td>0.9255</td>
<td>0.9232</td>
<td>0.9264</td>
</tr>
<tr>
<td>$\eta_{\text{itc}}$</td>
<td>0.9926</td>
<td>0.9937</td>
<td>0.9938</td>
</tr>
<tr>
<td>$\eta_{\text{aa}}$</td>
<td>0.9498</td>
<td>0.9496</td>
<td>0.9491</td>
</tr>
<tr>
<td>$\eta_{\text{ref}}$</td>
<td>0.8800</td>
<td>0.8800</td>
<td>0.8800</td>
</tr>
<tr>
<td>$\eta_{\text{year}}$</td>
<td>0.6401</td>
<td>0.6409</td>
<td>0.6430</td>
</tr>
<tr>
<td>Insolation Weighted Heliostat Field Efficiency</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\eta_{\text{year}, I}$</td>
<td>0.6897</td>
<td>0.6916</td>
<td>0.6933</td>
</tr>
<tr>
<td>Heliostat Field Area</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Area [x10^3m^2]</td>
<td>439</td>
<td>391</td>
<td>379</td>
</tr>
</tbody>
</table>

An advantage of the spiral pattern is that the density of heliostats better matches the pattern of heliostat efficiency as a function of position. Simply, the efficiency of the heliostat field is improved by placing more heliostats in a higher efficiency location of the field until of course the effect of shading and blocking exceeds the incremental benefit in the remaining efficiency terms, which is the reason the coefficient $b$ in Equation
required optimization instead of assuming the same value as for sunflowers. Figure 8(a) shows the layout of the redesigned PS10 plant with the radially staggered configuration. The color map indicates the efficiency of each heliostat position neglecting shading and blocking to illustrate how efficiency changes as a function of position on the heliostat field. Figure 8(b) represents the density of the heliostat field as a function of position by plotting the minimum distance between neighboring heliostats as a function of position. Finally, Figure 8(c) shows the efficiency of each heliostat position including shading and blocking for the given field. Inherent in the radially staggered configuration is a non-monotonic heliostat density as a function of distance from the receiver. This is clearly suboptimal because the heliostats at the high density regions are incurring significant shading and blocking while the heliostats in the low density regions are not fully utilizing the high efficiency field positions. On the other hand, in the spiral pattern shown in Figure 9, the density function is both continuous and monotonic. Moreover, from Figure 9(c) it is evident that the increase in heliostat density does not result in increased shading and blocking.

Figure 8: Comparison of Heliostat Efficiency and Density for the Improved Radially Staggered Configuration

Another benefit of the phyllotaxis spiral is that the polar angle of the heliostat locations, shown in Equation (14), is related to the irrational golden ratio $\phi$. Therefore, no two heliostat centers share the same azimuth angle with respect to the receiver tower, as opposed to the radially staggered pattern, where, within a given zone, every other row shares the same azimuth angle for every heliostat. Therefore, when blocking does occur in the spiral pattern, it is more likely to be localized.

Lastly, Table 4 presents the results of scaling the size of the field of PS10 to observe trends in efficiency and heliostat field area for larger plant sizes. At each heliostat field size, the parameters of the PS10 plant remain constant, except for the number of heliostats and size of the receiver aperture dimensions. Note
6 Conclusion

In this article, both a model and heuristic for heliostat field optimization are presented. The model, developed in object-oriented Fortran 95, is detailed herein and validated with the use of SoiTRACE [Wendelin(2003)]. As presented in Section 1, the development of the model offers opportunities to improve upon existing methods for calculating heliostat field efficiency including a heliostat discretization approach which is both fast and accurate in the calculation of shading and blocking and spillage. Additionally the model takes that a variation of the tower height would likely result in higher efficiency. The following function roughly calculates the diameter of the receiver aperture necessary to maintain the same interception efficiency as in PS10 for the optimized heliostat field as a function of the number of heliostats. This function is simply a polynomial fit of the annual average spot size at each of the field sizes and used to prevent high spillage losses, not the result of a detailed analysis in receiver design.

\[
d_a = -2.42 \cdot 10^{-7} n_h^2 + 5.37 \cdot 10^{-3} n_h + 8.95
\]  

As the number of heliostats is increased from the original 624, the difference in field efficiency between the optimized phyllotaxis spiral pattern and the radially staggered pattern grows while maintaining a roughly 18% reduction in heliostat field area. In other words, the proposed pattern outperforms the radially staggered pattern and the difference between the two is more pronounced at larger field sizes.
iterations throughout the day in steps of solar state which results in more instantaneous field efficiency evaluations around solar noon when the rate of change of solar azimuth is fastest and when insolation is greatest, thus producing a more accurate integration with fewer iterations.

The PS10 power tower plant in Andalusia, Spain, is used as a demonstrative application which both shows results similar to those available in the open literature [Wei et al.(2010)] and the ability of the model to improve upon existing configurations. As shown in Section 4, even with the existing heuristic of placing the heliostats in radially staggered pattern, the optical efficiency of the heliostat field can be improved by 0.19 percentage points while simultaneously reducing the land area by 10.9% simply by optimizing the parameters of the heuristic.

Finally, a new heuristic inspired by disc phyllotaxis [Vogel(1979)] is presented. Section 5 demonstrates the application of the spiral pattern by again redesigning the original heliostat field layout of PS10 for an improvement in optical efficiency of 0.36 percentage points and reduction in land area of 15.8%. Figures 8 and 9 illustrate the difference in radially staggered and spiral patterns’ heliostat density and the ability to utilize high efficiency land area. Note that herein maximization of efficiency and minimization of footprint was considered. A more important metric for practice is the levelized cost of energy (LCOE), which however depends on many uncertain economical parameters. However, the simultaneous increase of efficiency and decrease of land area achieved, directly implies a reduction in LCOE, irrespectively of the economic parameter values. While the spiral pattern performs better than traditional approaches, there is likely still room for significant improvement by considering other functional forms and heuristics.

As mentioned in Section 1, future work includes the optimization of unconventional sites for heliostat fields, such as hillsides in the CSPonD concept [Noone et al.(2011), Slocum et al.(2011)]. Without the simplifications used in planar layouts, hillsides are expected to rely more heavily on local optimization of individual heliostats and will require the automatic differentiation tools introduced with optimization algorithms to generate suitable heliostat configurations.

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