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1. Introduction

Time reverse modeling (TRM) is applied in several fields of science such as medical and earth sciences [1]. Prior time reverse studies focus on filtering single events out of recordings of low signal to noise (S/N)-ratio [2] or on spatial and temporal accuracy of single event localization [3]. We want to transfer a TRM approach within exploration geophysics to applications within the field of non-destructive testing (NDT). The fundamental problem is to identify a source location by solving the elastodynamic wave equation with limited data recorded at the boundaries of the modeling domain. However, those inversion problems are often ill-posed. We discuss application limitations and solution strategies.

Steiner et al. [4] applied TRM of the recorded surface wave field to better understand the passive low-frequency seismic wave field around hydrocarbon reservoirs and to determine whether some of the low-frequency signals originate from hydrocarbon reservoirs. The TRM methodology uses numerical algorithms for two-dimensional elastic wave propagation in combination with an imaging condition. This imaging condition is implemented because the measured low-frequency...
tremor signals were more or less continuous in time and no individual events or first arrival times could be detected. During TRM emitted seismic energy is propagated back to its origin. The seismograms recorded with synchronized three-component seismometers are reversed in time and implemented as sources for numerical wave extrapolation. The signals are propagated backwards through the velocity model. The backward propagated energy will focus at the source location, if the velocity model is accurate [3,4].

Concrete as a strongly heterogeneous and densely packed composite material represents a very important but also very difficult object for ultrasonic NDT methods. Due to the high density of scattering constituents and inclusions, ultrasonic wave propagation in this material consists of a complex mixture of multiple scattering, mode conversion and diffusive energy transport [5]. For a better understanding of the effect of aggregates, porosity and of crack distribution on elastic wave propagation in concrete and to optimize inverse reconstruction techniques, e.g. Impact echo methods [6], it is useful to simulate the wave propagation and scattering process explicitly in the time domain [7–9].

Acoustic emissions (AE) are caused by strain energy release due to irreversible processes such as cracking or internal friction in the material considered. Acoustic emission analysis (AEA) has become a promising method to evaluate the condition of concrete structures [10,11]. Qualitative procedures make use of basic parameters of recorded signals and try to identify the stage of degradation and to estimate the load history. Quantitative procedures attempt to identify characteristics of an AE source and therefore have to consider the wave propagation between source and sensors. Profiting from the methods of geophysics, considerable results have been achieved with respect to determination of onset times (picking), source localization and moment tensor analysis. Further progress depends on the handling of different crack distributions and cracked concrete itself as the medium for elastic wave propagation, considering also other elements of structural concrete such as reinforcement bars and prestressing tendons.

The paper aims at establishing a tool for localization and further evaluation of acoustic emissions in structural concrete and shall contribute to better understanding on the relation between crack growth and AE activity. Concrete pervaded by cracks serves as a medium for elastic wave propagation and shall be observed, described, interpreted and modeled. The procedure includes physical tests, to record and visualize crack formation and to obtain real AE data. Numerical forward modeling is used to evaluate and understand the physical tests. In this paper a feasibility study on a numerical concrete specimen is performed for this planned approach.

The challenging aspect for us is to transfer the time reverse modeling localization procedure by Steiner et al. [4] (described above) from geophysical exploration to the field of non-destructive testing in order to image acoustic emissions occurring during crack nucleation and growth in concrete. With TRM, receivers located like those in the physical model are considered as sources in the numerical simulation. AE are traced back in time, and the location of the generating fracture process will become visible as a concentration of elastic energy. Advantages of this method are the independency of detection/picking algorithms, the capability to handle low signal to noise ratios, and a possible identification of the moment tensor of the AE sources.

In the first part of the paper we briefly review the TRM approach by Steiner et al. [4]. In addition we perform a numerical accuracy test based on a 3D analytical solution. For the second part we generate a 2D numerical concrete model and determine the corresponding effective elastic properties. This allows a 2D plain strain feasibility study on the applicability of TRM in non-destructive testing. We conclude the paper with an outlook to 3D applications using real AE data.

2. Time reverse modeling of waves

In practise TRM is based on signals obtained from experiments which are send back into a specimen as sources in a time reverse manner. In this paper we investigate the potential of the TRM approach using a computational framework. This means that a forward simulation is used to produce signals with full control of their origin. The forward computation is typically based on an excitation localized in space and time. The waves emitted from this excitation are simulated up to a certain end time, while displacement values are recorded at specific boundary-points over time. These time-dependent values serve as signals in subsequent TRM simulations. Some mathematical details are described in this section. More details can be found in the work of Fink [12].

The computations of this paper are based on the linear anisotropic elastic wave equation for the displacement field \( u(x,t) \in \mathbb{R}^d \) (\( d = 2 \) in this paper) which reads:

\[
\rho \frac{\partial^2 u_i}{\partial t^2} = \delta_j \left( c_{ijkl} \partial^2 u_k + f_l \right) \quad \text{in} \quad \Omega \times [0,T].
\]

The spatial domain of interest is denoted by \( \Omega \subset \mathbb{R}^d \) with positions \( x \in \Omega \) and we consider time \( t \in [0,T] \) with some end time \( T \). We use index notation for vectors and tensors with summation convention and \( \partial_t = \partial/\partial t \). In general, the equation features an inhomogeneity \( f_l(x,t) \), which represents a space and time-dependent body force. The gravitational density is given by \( \rho_g(x) \) and \( c_{ijkl}(x) \) denotes the stiffness tensor, which gives the momentum tensor by \( m_{ij} = c_{ijkl} \xi^l u_k \). The equation contains no damping term, such that it is form-invariant with respect to time-inversion transformation \( t \mapsto -t \). For more information about the modeling, see for example [13]. Initial conditions are required for \( u_i \) and the velocity \( \partial_t u_i \), while boundary conditions can be of Dirichlet or Neumann type for \( u_i \).

The aim of TRM is to solve an inverse problem for the wave equation approximately. Note, that it is not the task to find a spatial and/or temporal distribution of a body force from boundary recordings, which is an ill-posed problem in general.
Instead the basis is an essentially homogeneous wave equation and the aim is to reconstruct a localized initial condition for the displacement field from recorded boundary values. We distinguish between two different TRM procedures: source time reverse modeling (source TRM) and full time reverse modeling (full TRM). An overview about these procedures is displayed in Fig. 1.

2.1. Forward computation

A standard forward computation based on (1) is utilized to produce boundary signals which enter TRM simulations. In this paper we always apply the so-called rotated staggered finite-difference scheme to discretize the wave equation. For a description of the numerical method see [14,15].

In our forward computations the initial displacement and velocity are not directly initialized but generated in the first time steps by a body force. Hence, the initial conditions are set to zero both for \( u_i \) and its velocity, while the body force is chosen to be:

\[
f_i(x, t) = \begin{cases} R_i(x, t) & t \in [0, t_s], \\ 0 & t > t_s, \end{cases}
\]

which vanishes for time \( t > t_s \) with a start-up time \( t_s \ll T \). Typically, \( R_i \) is chosen localized in space around a position \( x \), with a specific excitation pattern for example a second derivative of a Gaussian. After time \( t_s \) a localized non-vanishing displacement field is generated which can be considered as actual initial condition emitting waves towards the boundaries. To implement a free surface on the boundary of \( \Omega \) Neumann conditions for \( u_i \) are used. The aim of the TRM simulation below is to find an approximation to the original source position \( x_s \).

2.2. Source time reverse modeling

During a forward computation values of displacement are recorded by receivers on the boundary \( \partial \Omega \) of the specimen. The locations of the receivers are denoted by

\[
S = \{ x^{(1)}, x^{(2)}, \ldots, x^{(N)} \} \subset \partial \Omega,
\]

where \( N \) is the total number of source positions. In the following we will typically use \( N = 12 \). The time series of the displacement at position \( x^{(k)} \) is written:

\[
u_i^{(k)}(t) = u_i(x^{(k)}, t),
\]

with time \( t \in [0, T] \). These time series serve as input data for a TRM simulation. The position arrangement can be varied to evaluate the reproduction ability of the TRM simulation.

The TRM simulation is again based on the wave Eq. (1) using the same coefficients from the forward computation as well as \( x \in \Omega \) and \( t \in [0, T] \). No body force is present throughout the computation, \( f_i = 0 \). Initial conditions for \( u_i \) and \( \partial_t u_i \) are vanishing, such that the equation is driven by boundary conditions. On \( \partial \Omega \) the recorded signals \( u_i^{(k)} \) are fed as sources into the domain. Formally, we write:

\[
u_i(x, t) = u_i^{(k)}(T - t) \quad \text{for} \quad x \in S \subset \partial \Omega
\]
\[
u_i(x, t) = 0 \quad \text{for} \quad x \in \partial \Omega \setminus S,
\]

such that inhomogeneous Dirichlet data is given exclusively in the source locations \( S \). Note, that the time series is fed into the computation backwards in time. Hence, the TRM simulation reverses the forward computation.

The term source TRM emphasizes the way the time signals are implemented in the algorithm, i.e. as sources of wave excitations. source TRM is not complete by definition in the sense that the equation is provided with time-reversed recei-

![Fig. 1.](image-url)
ver-signals at every boundary-point. Only a few selected points are used. In brief, we do not provide the equation with the full set of information. It turns out that only a few boundary-points have to be provided with time-reversed signals to achieve very good results. Note that the source TRM has also been applied successfully to real models, i.e. using receiver data (representing the forward simulation) to carry out a numerical time reverse simulation in order locate a real physical wave exciting source. Such a real data example within exploration geophysics can be found in [4].

In the numerical method the actual domain \( \Omega \) is supplemented by a layer of almost vacuum with zero Dirichlet conditions at the outer computational boundary as described above. Hence, the time signals \( u_i^{(k)} \) are inserted inside the numerical grid on the boundary grid points \( \partial \Omega \) of the medium specimen. To avoid scattering they are superimposed to any existing values at these grid points that are the results of interior and surface waves. By this technique the signals are interpreted as time series of localized initial conditions whose evolutions are superimposed in a time-delayed manner.

The waves emitted from the boundary sources during a source TRM simulation will interfere constructively in the displacement field and the location of strongest interference is taken as an approximation of the source location \( x_0 \) of the original initial condition. In order to easily display the result of this interference in a TRM simulation we introduce the so-called TRM-field defined by

\[
\text{TRM}(\mathbf{x}) := \max_{t \in [0,T]} ||u_\mathbf{x}(\mathbf{x}, t)||, \tag{7}
\]

for every point \( \mathbf{x} \in \Omega \). This means, in order to image the convergent wave focusing on the initial source, we store the maximum particle displacement for each grid point throughout the entire time of modeling. The highest value of the TRM-field finally makes it possible to locate the original source location, i.e. the source location of the forward simulation.

### 2.3. Full time reverse modeling

Due to time invariance an initial condition for a homogeneous wave equation can be exactly recovered in a time-reversed computation from the time series of all boundary values for times \( t \in [0,T] \) and the displacement field and its velocity at time \( T \). An approximation to the initial conditions can be found when only the displacement field \( u^{\text{fwd}}(\mathbf{x}, T) \) and its velocity \( \partial_t u^{\text{fwd}}(\mathbf{x}, T) \) at time \( T \) are given from a forward simulation but no knowledge of the boundary values is available.

For such a time reverse simulation we feed in the final fields as initial conditions:

\[
u_i(\mathbf{x}, t = 0) = u_i^{\text{fwd}}(\mathbf{x}, T) \quad \text{in } \Omega, \]

\[
\partial_t u_i(\mathbf{x}, t = 0) = -\partial_t u_i^{\text{fwd}}(\mathbf{x}, T) \quad \text{in } \Omega, \tag{8}
\]

where the velocity is taken negative in correspondence to the time reversal. Since the numerical method is based on a two-step time integrator, see [14], such a reversed computation is easy to realize from the last two displacement fields at time \( T \) and \( T - \Delta t \). Both fields are simply used as initial fields for the two-step integrator in a reversed order. Boundary conditions are the same as in the forward computation.

The result of such a full TRM simulation can be viewed as benchmark for a source TRM since much more information is used and typically the localization of the initial excitation is much better, see the example in the next section. However, full TRM is typically not usable in practice due to the lack of knowledge in realistic situations.

### 2.4. Example for comparison

To illustrate the different procedures we will compare full TRM to source TRM for a generic example. We created a homogeneous 2D model surrounded by a thin vacuum layer. The model (grid spacing \( h = 0.0001 \) m consists of a \( 10 \times 10 \) cm area in which the compressional and shear wave velocity is set to \( v_p = 3987 \) m/s and \( v_s = 2328 \) m/s; the density is \( \rho_g = 2376 \) kg/m\(^3\). An initial body force source (second derivative of a Gaussian with \( f_{\text{fund}} = 100 \) kHz) in horizontal direction is placed at (400,300) and marked with a white circle (Fig. 2(a)). The modeling is done with second order time update and a second order spatial differentiation operator as described in [14] using a time step \( \Delta t = 1.8 \times 10^{-8} \) s. The full wave field (i.e. the vertical and horizontal displacement field) is recorded at 12 sensor positions (marked with crosses in Fig. 2(a)) during the full length of the simulation. In addition, the complete wave field is stored at two consecutive time steps at the end. This is the necessary input data for the source TRM and full TRM, respectively.

Fig. 2(b)) shows the TRM-field which results from the full TRM simulation. It accurately shows largest values around the location of the initial source. The TRM-field of the source TRM simulation is displayed in Fig. 2(c). Remarkably, the initial source location is also accurately recovered.

### 2.5. TRM source patterns

In NDT it is relevant not only to detect the initial source localization but also the excitation pattern in the momentum tensor. To simulate different excitations we model the external force \( f_i \) of the forward computation (2) by

\[
f_i(\mathbf{x}, t) = \partial_t \left( m_i^{\text{ex}}(\mathbf{x}, t) \right), \tag{10}\]
that is, derived from a given momentum tensor $m^{\text{ex}}_{ij}$. We want to briefly demonstrate that it is possible to extract the specific form of $m^{\text{ex}}_{ij}$ from a time reverse computation with only few source inputs on the boundary.

In the example of (2) above we used a body force source in horizontal direction. This force has been modeled by a moment tensor with vanishing entries except for the $m_{11}$ component which was given by a localized wavelet $w(x,t)$ localized around the source location $x_s$. We investigated two more different source types namely an explosion and an arbitrary moment tensor. The definitions are given by

(a) **Horizontal force**: $m_{11} = w(x,t), m_{22} = m_{12} = m_{21} = 0$

(b) **Explosion**: $m_{11} = m_{22} = w(x,t), m_{12} = m_{21} = 0$

(c) **Arbitrary**: $m_{12} = m_{11} = m_{22} = w(x,t), m_{11} = 0$

The TRM-field for the horizontal force has been already shown in Fig. 2(b) and is enlarged in Fig. 3(a). However, different initial excitation types will give different source patterns. In Fig. 3(b) and (c) we show the pattern for an explosion and the arbitrary source with the same source wavelet $w(x,t)$, respectively. It can be observed that the TRM-field is able to distinguish between different excitation characteristics. A catalogue of typical TRM-fields from generic excitations and a time-dependent analysis of the stress-field will be useful to identify the excitation in realistic TRM simulations.

### 2.6. 3D analytical forward solution

Two-dimensional (2D) models have the advantage that they require relatively modest computing power in comparison to three-dimensional (3D) ones. Most situations in nature, however, are three-dimensional, so that two-dimensional simulations represent a somewhat artificial model of the real situation. More precisely, our 2D setup implies a 2D plain strain boundary condition. This section discusses a specific effect: vertical offset of source localization. The simulations we carried
out were motivated by observations made for 2D time reverse simulations with a receiver line placed directly above the
known source and with other receiver lines positioned in a certain horizontal distance away from the first line. It was ob-
served that the TRM-field of maximum particle displacement localized the source in different depths depending on the re-
ceiver line chosen for the 2D time reverse simulation.

In order to understand those effects, we use a 3D analytical solution of the elastodynamic wave equation for a point force
in a homogeneous, isotropic and unbounded medium as described in Aki and Richards [13].

If a point force \( \mathbf{f} \) in direction \( e_i \) with general time-varying amplitude \( X_0(t) \) acts at a particular fixed point \( O \) which we
choose to be the origin of a fixed Cartesian coordinate system and if the medium is elastic, isotropic, homogeneous and un-
bounded, the displacement \( u_i(x, t) \) at point \( x \) and time \( t \) in direction \( e_i \) is given by the following explicit formula:

\[
u_i(x, t) = \frac{1}{4\pi\rho} \left(3\gamma_i' \gamma_j - \delta_{ij} \right) \int \frac{t X_0(t - \tau) d\tau}{r^3} + \frac{1}{4\pi\rho}\gamma_j \int \frac{t X_0(t - \tau) d\tau}{r^2} - \frac{1}{4\pi\rho}\gamma_j \int \frac{t X_0(t - \tau) d\tau}{r},
\]

where the following notations have been used:

- \( r := |x| \) denoting the distance of point \( x \) from the point, where the point force \( \mathbf{f} \) is acting.
- \( \gamma_i := x_i/r \) \((i = 1, 2, 3)\) denoting cosine of direction.
- \( \alpha, \beta \) and \( \rho \) denoting P-wave, S-wave velocities and density of the medium.
- \( \delta_{ij} \) is Kronecker’s delta.
- \( \text{NF} \) stands for the near field part of the solution.
- \( \text{FFP} \) stands for the far field of the compressional (\( P- \)) wave part of the solution.
- \( \text{FFS} \) stands for the far field of the shear (\( S- \)) wave part of the solution.

A vertical-force source with the same characteristic as used for the forward simulation described above is used
\( (f_{\text{fund}} = 100 \text{ kHz}, \Delta t = 1.8 \times 10^{-8} \text{ s}, \text{second derivative of a Gaussian}) \). The homogeneous medium parameter of this example was used as well \( (\nu_s = 3987 \text{ m/s} \) and \( \nu_t = 2328 \text{ m/s} \); the density is \( \rho = 2376 \text{ kg/m}^3 \)). There are three important differences
to the numerical forward simulation described in Section 2.5. First, the analytical solution is a full 3D solution of the elas-
todynamic wave equation. Therefore we also obtain analytical forward receiver data with an offset to the plane including
the source (Line 1 in Fig. 4(a)). Second, the analytical solution is available for an unbounded homogeneous elastic medium
only. Third, the receivers are placed in lines only in one direction away from the source position. This non-symmetric con-
figuration will influence the localization accuracy (shown below). The used model size is typical for engineering applications
but the results can be easily transferred to geophysical problems.

The 2D source TRM series of simulations are based on the setup shown in Fig. 4(a), which consists of 7 lines with 12
receivers placed in a vertical distance of 337 grid units from the source. The length of the individual receiver lines is five
times as large as the distance from the source to the first receiver line. Receiver line 1 is placed directly vertical above
the source, receiver line 2 is located 50 grid units in y-direction from receiver line 1 and so on. In order to comply with
the condition unbounded medium given by the analytical solution, we chose the dimension of the numerical grid in the time
reverse simulation as large that the P-wave would not reach the edge of the model during the simulation.

First, we calculated the receiver data of receiver line 1 using the 3D analytical solution. We then run the two-dimen-
sional source TRM simulation based on this set of data. This procedure was successively repeated for the remaining lines. Fig. 5

![Fig. 4](image-url) The setup for the 3D numerical forward simulation is shown on the left hand side. The determined source depth versus the offset is displayed on the right hand side.
shows the 2D-TRM-field resulting on the plane directly under the corresponding receiver line. The white square marks the location of the original source and the black cross the maximum value.

It is visible that the location of the maximum value shifts vertically downwards with increasing offset of the receiver line relative to the source location (Fig. 4(b)). We display the offset of the receiver lines on the x-axis and the depth indicated by the corresponding TRM-fields on the y-axis. We observe a clear trend to deeper depths the farther a receiver line is located from the original source. This can be explained by the non-symmetric distribution of the receivers around the source in combination with the non-zero wavelength of the propagating elastic waves. However, for most practical applications the observed localization accuracy is acceptable. More generally, we want to point out that the analytical forward solution presented here can also used for an analytical TRM approach in an unbounded homogeneous medium [16].

3. NDT-application: localization of acoustic emissions in concrete
3.1. Numerical concrete model

A numerical concrete model in 2D with randomly distributed concrete constituents similar to [17] is presented (Fig. 6(a)). The model displays an arbitrary cross section of the numerical concrete specimen. Plain strain is assumed for each cross section. The concrete specimen is simplified by spatial randomly distributed ellipses (grains) and circles (air voids) and filled between with homogeneous cement paste. The aggregates and air voids are effectively modeled as infinite cylinders. The grain-size distribution is transferred from a real concrete mix and is in agreement with Fuller’s curve [18]. Air inclusions are estimated with 2%, a common limit percentage in practice. The elastic material properties for the grains ($v_p = 4180$ m/s, $v_s = 2475$ m/s, $\rho = 2610$ kg/m$^3$), the air voids ($v_p = 0$ m/s, $v_s = 0$ m/s, $\rho = 0.0001$ kg/m$^3$) and for the cement paste ($v_p = 3950$ m/s, $v_s = 2250$ m/s, $\rho = 2050$ kg/m$^3$) are allocated to grid points for the finite-difference algorithm (grid spacing $h = 0.0001$ m). For reasons of verification computer tomography (CT) screens of a concrete cube (Fig. 6(b)) are provided. Com-

![Fig. 5. TRM-field for Line 1 where the receivers were located directly above the point source (grid spacing $h = 0.0001$ m). The time-reversed signals, determined by an analytical solution, are inserted at 12 points marked with a white circle. The cross is the maximum value of the TRM-field. There is a vertical localization error with respect to the original source location (white quadrat).](image)

![Fig. 6.](image)

Fig. 6. Numerical concrete model (10 x 10 cm) versus CT screen of concrete specimen (12 x 12 cm). Displayed are normalized density values. Dark colors correspond to air inclusions, grey to the cement paste and light color defines the grains.
parison of properties of the introduced numerical concrete model versus the CT model (real concrete) shows a good agreement and suggests its effectiveness and applicability to non-destructive testing of concrete. Moreover, the numerical concrete model can be straightforward varied with different grain size and air void distributions for further parameter studies. However, it is also possible to use the CT screens as input for our numerical simulations. This is illustrated in the section ‘outlook’. We choose here the numerical concrete model because, (1) it can be generated with a full control of the grain-size distribution, and (2) we want to avoid artefacts due to segmentation errors. Segmentation of raw CT data is one difficult and necessary processing step (e.g. [19] and references therein).

3.2. Forward simulation with an arbitrary chosen momentum tensor source

In order to create a synthetic but realistic data set we use the following model setup. An arbitrary chosen momentum tensor source with source time-function as described in Section 2.5 is used. The discretization details are adopted from the previous section. In the snapshot shown (Fig. 7(a)) a wave field excited at (400, 300) is illustrated. Due to the heterogeneous concrete model a lot of scattering can be observed compared to the wave field in Fig. 2(a). Two-components (horizontal and vertical displacement) of the emitted waves are recorded at 12 sensor positions of sensors marked by black crosses on the boundaries.

3.3. Reverse simulation with 12 sensors using exact velocity model

The time reverse computation is executed for two sensor modifications, sensors with two components (displacement normal and parallel to surface) and sensors with one component (displacement normal to surface) recorded. Steiner et al. [4] applied TRM successfully on data measured with three-components seismometers. For most NDT applications (i.e. for piezoelectric sensors) the case where one-component data is recorded on sensor positions is relevant. The performance is analyzed considering two-component sensor modification and discussed. As described before by using both components the time-reversed propagating waves focus on the source coordinates they originated from. An excellent result is obtained (Fig. 7(b)) in that the radiation pattern of the induced source can be visual identified (marked by the white circle) by means of the characteristic according to Fig. 3(c). Using only the displacement component normal to concrete surface a good focus can be observed (Fig. 7(c)). The radiation pattern seems blurred, but represents a satisfying achievement if only one-component data used for reverse computation is considered.

3.4. Effective elastic properties of the used numerical concrete sample

To obtain effective velocities of the numerical concrete sample (Fig. 6(a)) we use a technique described in detail in [20]. A review of this and related methods is given in [21]. We apply a body force plane source at the top of the model. The plane wave generated in this way propagates through the numerical concrete model. With two horizontal planes of receivers at the top and at the bottom, it is possible to measure the time-delay of the peak amplitude of the mean plane wave caused by the inhomogeneous region. With the time-delay (compared to a homogeneous reference model) one can estimate the effective velocity of the compressional and shear wave. The source wavelet in our experiments is always the first derivative of a Gaussian with a dominant frequency of 12500 Hz and with a time increment of $\Delta t = 1.8 \times 10^{-9}$ s. As a result, we have determined the effective compressional wave velocity to $v_{p,\text{eff}} = 3987$ m/s and the effective shear wave velocity to $v_{s,\text{eff}} = 2328$ m/s.
3.5. Reverse simulation with 12 sensors using effective elastic properties for concrete

Given the effective elastic properties (EEP) with \( v_{p,\text{eff}} \) and \( v_{s,\text{eff}} \) a reverse computation is executed similar to Section 3.3. The forward simulation is performed based on the heterogeneous medium as described in Section 3.2, but for time reverse modeling the previous determined EEPs are used instead. Considering both sensor components a very good result can be achieved (Fig. 8(a)). In the backward propagation direction no scattering will occur and compared to reverse simulation with the exact velocity model (Fig. 7(a)) a convergence of elastic energy can be observed. The TRM source pattern is clearly visible. Using only the displacement component normal to the concrete surface a problematic result can be obtained (Fig. 8(b)). Artifacts due to surface waves are significantly visible around the boundaries and the source pattern is displayed rudimentary. Possible solutions to improve the localization are to place more sensors on the surface and to magnify the energy induced into the model and to interpolate the zero values on the boundary between sensor positions with respect to the incoming waves hitting the boundary. An irregular sensor arrangement may be investigated to improve the performance of the method and to clarify their influence on the wave focus.

4. Outlook

The forthcoming approach aims at extending the introduced two-dimensional TRM method to three dimensions. We show the first steps of this approach.

A concrete specimen screened in thin slices (Fig. 9(a)) is considered and after post processing (e.g. threshold-segmentation) visualized as a complete three-dimensional model (Fig. 9(b)). The digital format is beneficial for further considerations such as simulations of elastic wave propagation.
The segmented CT data can be read and translated into a feasible format for wave propagation computations on a FD grid. Elastic material properties such as p-wave velocity $c_p$ and S-wave velocity $c_s$ are allocated to the segmented aggregates. Numerical simulations starting a double-couple source excitation (Fig. 9(c)) are performed on the high performance parallel computing cluster at the central computing facilities of ETH Zurich. The influence of density distribution of aggregates, air voids percentage and crack distribution on elastic wave propagation will be investigated separately. The numerical results are compared to data obtained from physical tests and are to be discussed.

The TRM method presented in this paper was transferred from exploration geophysics (km-scale) to acoustic emission analysis of a single concrete sample (cm-scale). Therefore we assume that this technique can also be transferred to other NDT applications such as tubes inspection in nuclear power plants or gas-pipe inspection buried under ground. Most important is that the discussed method is able to localize a (secondary) source of acoustic waves. This can be for example a crack which can scatter an elastic wave in a thin plate of steel (plain strain). However, for such an application we recommend to perform numerical feasibility studies as presented in this work.

5. Conclusion

Time reverse modeling using the elastodynamic wave equation is, due to the increasing computational possibilities, nowadays fast and accurate. We used the rotated staggered FD grid to calculate effective elastic properties of concrete. Our numerical modeling can be considered as an efficient and well-controlled computer experiment. The numerical simulations show that source areas and characteristics of acoustic emissions can be located using TRM. With our feasibility study we demonstrate that our approach is ready to be applied in the laboratory for a deeper understanding of experiments in the area of non-destructive testing. We have demonstrated that with a limited number of sensors and an effective homogeneous elastic model the accuracy of localization is acceptable.

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